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Electron transport in an open mesoscopic metallic ring

Dibyendu Roy

Raman Research Institute, Bangalore 560080, India

E-mail: dibyendu@rri.res.in

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Abstract

We study electron transport in a normal metallic ring modelled by a tight-binding lattice Hamiltonian, coupled to two electron reservoirs. First, Büttiker's model of incorporating inelastic scattering, and hence decoherence and dissipation, has been extended by connecting each site of the open ring to one-dimensional leads for uniform dephasing in the ring threaded by a magnetic flux. We show with this extension that conductance remains symmetric under flux reversal, and Aharonov–Bohm oscillations with changing magnetic flux reduce to zero as a function of the decoherence parameter, thus indicating dephasing in the ring. This extension enables us to find local chemical potential profiles of the ring sites with changing magnetic flux and the decoherence parameter in an analogous way to the four-probe measurement. The local electrochemical potential oscillates in the ring sites because of quantum-interference effects. This predicts that the measured four-point resistance also fluctuates and even can be negative. Then we point out the role of the electronic eigenstates of the closed ring in the persistent current around Fano antiresonances of an asymmetric open ring, for both ideal leads and tunnel barriers. Determining the real eigenvalues of the non-Hermitian effective Hamiltonian of the ring, we show that there exist discrete bound states in the continuum of scattering states for the asymmetric ring even in the absence of magnetic flux. Our approach involves quantum Langevin equations and non-equilibrium Green's functions.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The persistent current in equilibrium and the Aharonov–Bohm (AB) oscillations of conductance with changing magnetic flux, realized in a normal metallic ring, are two important achievements of mesoscopic physics. Büttiker *et al* [1] predicted the presence of persistent current in a closed normal metallic ring threaded by a magnetic flux ϕ in the coherent regime. The magnetic flux breaks down the time reversal symmetry of the Schrödinger equation and hence there exists a persistent current whenever the flux ϕ is not equal to a multiple of $\phi_0/2$ where ϕ_0 is the universal flux quantum. Gefen *et al* [2] connected two current leads to such a one-dimensional ring and calculated the conductance $G(\phi)$ between the two leads from the Landauer formula. The conductance shows AB-like oscillations with changing magnetic flux ϕ with period ϕ_0 because of interference of the electron wavefunctions coming through the two branches of the ring at the lead. Another kind of AB effect with principal period $\phi_0/2$ is present

in the ring because of interference of time reversed paths encircling the ring. These oscillations persist even when strong elastic scattering is present in the ring. Both the persistent current [3, 4] in a closed ring and the AB oscillations of conductance [5] in an open ring were experimentally realized at a temperature of a few millikelvin.

In real systems inelastic scattering is always present because of electron–phonon interactions [6] above about 1 K, whereas electron–electron interactions are expected to play the dominant role at low temperatures in the absence of extrinsic sources of decoherence such as magnetic impurities. Certainly inelastic scattering introduces decoherence, and both the above phenomena are diminished. Büttiker [7–10] proposed a phenomenological model of inelastic scattering, and hence dissipation and dephasing in the ring. This model is quite similar to the self-consistent reservoirs model, introduced by Bolsterli *et al* [11, 12] in the context of heat transport. In Büttiker's model, the ring is connected to a reservoir of electrons of chemical potential μ whose value is determined

self-consistently by demanding that the average electron current from the ring to this side reservoir should be zero. This conserves the total number of electrons in the original system. In this model the side reservoir destroys the coherence of conducting electrons by removing them from the transport channel and then re-injecting them in the channel with a different phase and energy; thus dephasing and dissipation can both occur. With a single Büttiker probe, the conductance of the open ring enclosing a magnetic flux satisfies the Onsager reciprocity relation, i.e. $G(\phi) = G(-\phi)$. But in this model dephasing occurs locally in space whereas in a realistic system it happens uniformly throughout the ring. There is another popular model [13] that incorporates dephasing, in which a spatially uniform imaginary potential is added in the Hamiltonian of the system which again removes electrons from the phase coherent transport channel. This model suffers from the drawback in that it violates the above stated Onsager reciprocity relation. Brouwer and Beenakker [14] removed the shortcomings in the imaginary potential model by re-inserting the carriers into the conducting channel to conserve particles. Then they compared the two above stated models for dephasing in a chaotic quantum dot. We also emphasize that they consider a single but multi-channel voltage probe, so a more careful formulation of uniform dephasing with voltage probes is clearly desirable.

Here we do a simple extension to get uniform dephasing in the ring with Büttiker probes. All the sites of the ring modelled by the tight-binding Hamiltonian are connected with one-dimensional electron reservoirs which are also modelled by the tight-binding Hamiltonian. Two distant side reservoirs with fixed chemical potential μ_L and μ_R act as source and drain, respectively. The chemical potentials of the other reservoirs are fixed self-consistently by imposing the condition of zero current. Now in this extended model decoherence occurs uniformly throughout space. We show that again the conductance $G(\phi)$ is symmetric under flux reversal, and the AB oscillations of $G(\phi)$ decay to zero as the strength of coupling, γ' , between the side reservoirs and the ring is increased. One nice consequence of this extension is that we can find the exact chemical potential profiles of the ring's sites by changing the magnetic flux by tuning the coupling γ' to almost zero. This is similar to a four-terminal resistance measurement with non-invasive voltage probes [15].

Persistent current in an open ring is realized even without any magnetic flux in the presence of a transport current [16, 17]. Two electron reservoirs with different chemical potentials are coupled with a mesoscopic ring in such a way that the lengths of the two arms of the ring between two contacts are different. A circulating current flows through the ring around certain Fermi energy values where the total transmission coefficient between two contacts goes to zero. We show here that at these antiresonance energy values there exist bound states in the continuum of scattering states (BIC) for the case of ideal leads. For single channel transport the energies of the bound states are exactly the same as those of the electronic eigenstates of the closed ring. We also discuss this issue for tunnel barriers.

We use the formalism introduced by Dhar, Shastry and Sen recently in two papers [18, 19]. They derived both the

Landauer results and, more generally, the non-equilibrium Green's function (NEGF) results on transport using the quantum Langevin equations approach. It is numerically easier to deal with the multiple reservoirs and disorder with this approach.

The outline of the paper is as follows. First, in section 2, we define the general model and describe how we get different current expressions in the linear response regime using the quantum Langevin approach. In section 3 we solve the extended Büttiker model for uniform dephasing. Next, in section 4, we discuss the issues regarding the persistent current in an open asymmetric ring and bound states in a continuum. Finally we conclude with a discussion in section 5.

2. Model and general results

We consider a one-dimensional mesoscopic ring modelled by the tight-binding lattice Hamiltonian. Two distant sites 1 and M of the ring are connected to two infinite reservoirs with specified chemical potentials μ_1 and μ_M . They are, respectively, source and drain. Each arm of the open ring between these two contacts has N_1 and N_2 sites, each of which is coupled to an infinite reservoir at chemical potential μ_l and small finite temperature T (see figure 1). All the reservoirs are also modelled by a one-dimensional tight-binding Hamiltonian. The total Hamiltonian of the system consisting of the ring and all the reservoirs is given by

$$\mathcal{H} = \mathcal{H}_r + \sum_{l=1}^N \mathcal{H}_R^l + \sum_{l=1}^N \mathcal{V}_{rR}^l,$$

where

$$\begin{aligned} \mathcal{H}_r &= - \sum_{l=1}^N \gamma (e^{-i\theta} c_l^\dagger c_{l+1} + e^{i\theta} c_{l+1}^\dagger c_l) \\ \mathcal{H}_R^l &= -\gamma_l \sum_{\alpha=1}^{\infty} (c_\alpha^{l\dagger} c_{\alpha+1}^l + c_{\alpha+1}^{l\dagger} c_\alpha^l) \quad l = 1, 2 \dots N \\ \mathcal{V}_{rR}^l &= -\gamma_l' (c_1^{l\dagger} c_l + c_l^\dagger c_1^l) \quad l = 1, 2 \dots N. \end{aligned} \quad (2.1)$$

Here c_l and c_α^l denote, respectively, electron annihilation operators on the closed ring and on the l th reservoir. Due to the periodic geometry of the ring, $c_l = c_{l+N}$ and contribution of magnetic flux ϕ have been included in $\theta = \frac{2\pi\phi}{N\phi_0}$. The Hamiltonian of the ring is denoted by \mathcal{H}_r , that of the l th reservoir by \mathcal{H}_R^l and the coupling between the ring and the l th reservoir is \mathcal{V}_{rR}^l . The parameters γ_l' control the hopping of electrons between reservoirs and the ring. Also the total number of sites in the ring $N = N_1 + N_2 + 2$.

Following [19, 20], we get the steady state solution of the ring variables in the Fourier domain,

$$\tilde{c}_l(\omega) = \sum_{m=1}^N G_{lm}^+(\omega) \tilde{\eta}_m(\omega), \quad (2.2)$$

where

$$\tilde{c}_l(\omega) = (1/2\pi) \int_{-\infty}^{\infty} dt e^{i\omega t} c_l(t), \quad G^+ = \frac{\hbar}{\gamma} Z^{-1},$$

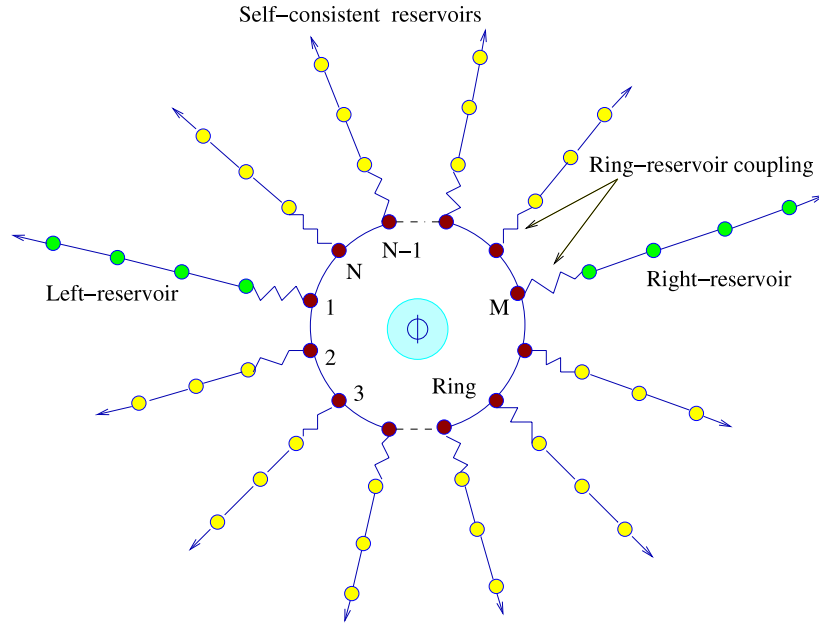


Figure 1. A schematic description of the model.

and

$$Z_{lm} = \frac{\hbar}{\gamma} (\omega - \Sigma_l^+) \delta_{lm} + e^{-i\theta} \delta_{l,m-1} + e^{i\theta} \delta_{l,m+1} + e^{i\theta} \delta_{l1} \delta_{mN} + e^{-i\theta} \delta_{lN} \delta_{m1}.$$

$G^+(\omega)$ is the Green's function of the full system (ring and reservoirs) and for points on the ring can be written in the form $G^+(\omega) = [\omega - H_r/\hbar - \bar{\Sigma}^+]^{-1}$ where $\bar{\Sigma}^+$, defined by its matrix elements $\bar{\Sigma}_{lm}^+ = \Sigma_l^+ \delta_{lm}$, is a self-energy term modelling the effect of infinite reservoirs on the isolated single particle ring Hamiltonian H_r . $\Sigma_l^+(t) = (\frac{\gamma'_l}{\hbar})^2 g_{1,1}^{l+}(t)$ where $g_{1,1}^{l+}(t)$ is the single particle Green's function of the l th reservoir at site 1. Here $\tilde{\eta}(\omega)$ is the noise characterizing reservoir's initial distribution. The effective ring Hamiltonian is $H_r + \hbar \bar{\Sigma}^+$, which can be shown to be non-Hermitian. We will use it to find bound states in a later section. Now one important point to notice is that, for θ not equal to an integral multiple of π , Z_{lm} is not a symmetric matrix. So the presence of magnetic flux ϕ breaks down the symmetric property of $G^+(\omega)$ whenever ϕ is not equal to an integral multiple of $N\phi_0/2$. This is a consequence of the loss of the time reversal symmetry of the problem in the presence of magnetic flux.

In the present work we are interested in the electron current from the reservoirs to the ring and also the current in the ring. For this purpose we first define the electron density operator on the ring sites and then use the continuity equation to get the corresponding current operators. Let us define j_l as the electron current between sites $l, l+1$ on the ring and j_{r-l} as the electron current from the ring to the l th reservoir. These are given by the following expectation values:

$$j_l = \frac{ie\gamma}{\hbar} \langle e^{i\theta} c_{l+1}^\dagger c_l - e^{-i\theta} c_l^\dagger c_{l+1} \rangle$$

$$j_{r-l} = \frac{-ie\gamma'_l}{\hbar} \langle c_l^\dagger c_l^l - c_l^l c_l^\dagger \rangle$$

where e is the charge of the electron. Using the general solution in equation (2.2) and the noise-noise correlation [20], we can do the above averaging and find

$$j_l = \sum_{m=1}^N \frac{-1}{2\pi} \int_{-\infty}^{\infty} d\omega \mathcal{F}_{lm} (f_l - f_m) \quad (2.3)$$

$$j_{r-l} = \sum_{m=1}^N \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \mathcal{T}_{lm} (f_l - f_m) \quad (2.4)$$

with

$$\mathcal{F}_{lm} = \frac{2\pi ie\gamma\gamma_m'^2}{\hbar^3} (e^{i\theta} G_{lm}^+ G_{m+1}^- - e^{-i\theta} G_{l+1m}^+ G_{ml}^-) \rho_m$$

and

$$\mathcal{T}_{lm} = \frac{4\pi^2 e\gamma_l'^2 \gamma_m'^2}{\hbar^4} |G_{lm}^+|^2 \rho_l \rho_m,$$

where $G_{ml}^- = G_{ml}^{*+}$ and f_l is the Fermi function. The chemical potentials of the reservoirs at the sites of the ring 1, M are specified by $\mu_1 = \mu_L$ and $\mu_M = \mu_R$. Here we restrict ourselves to low temperature and the linear response regime where the applied chemical potential difference $\Delta\mu = \mu_R - \mu_L$ is small, i.e. $\Delta\mu \ll \mu_{L,R}$ and $k_B T \ll \mu_{L,R}$. For notational simplicity we choose $\gamma_l = \gamma$ for $l = 1, 2, \dots, N$ and $\gamma'_l = \gamma'$ for $l = 2, 3, \dots, M-1, M+1, \dots, N$. With this assumption, the reservoirs including the source and drain will have the same Green's function and density of states and we will use the notation $g_{1,1}^{l+}(\omega) = g^+(\omega)$ and $\rho_l(\omega) = \rho(\omega)$ [20].

In the linear response regime, taking the Taylor expansion of the Fermi functions $f(\omega, \mu_l, T)$ about the mean value $\mu = (\mu_L + \mu_R)/2$, equations (2.3) and (2.4) reduce to the following set of equations:

$$j_l = \frac{-1}{2\pi\hbar} \sum_{m=1}^N \mathcal{F}_{lm} (\mu_l - \mu_m) \quad (2.5)$$

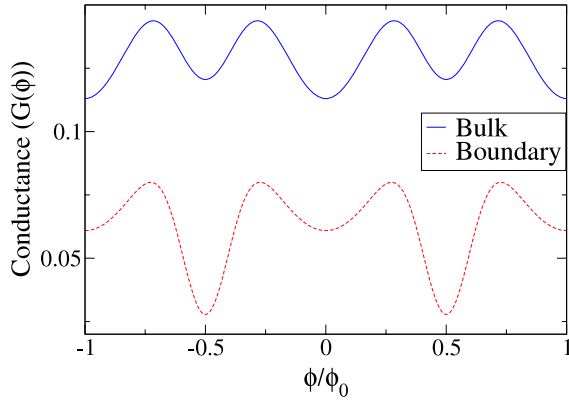


Figure 2. Plot of the conductance $G(\phi)$ of an open symmetric ring with a single Büttiker probe. The total number of sites in the ring $N = 20$ and $\gamma' = 1.5$.

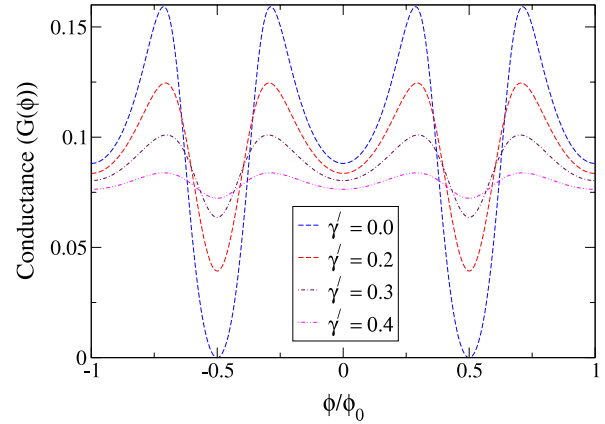


Figure 3. Plot of the AB oscillations of conductance $G(\phi)$ of the open symmetric ring with uniform dephasing for different strengths of coupling γ' , with $N = 20$.

$$j_{r-l} = \frac{1}{2\pi\hbar} \sum_{m=1}^N \mathcal{T}_{lm} (\mu_l - \mu_m) \quad \text{for } l = 1, 2 \dots N, \quad (2.6)$$

where \mathcal{F}_{lm} and \mathcal{T}_{lm} are evaluated at $\omega = \mu/\hbar$. These are linear equations in $\{\mu_l\}$ and are straightforward to solve numerically. In the next section we will consider the case of an open ring in the presence of uniform dephasing and dissipation. Later, we will study the persistent current in an asymmetric open ring in the absence of both magnetic flux and decoherence by external reservoirs.

3. Extended Büttiker model for uniform dephasing in an open ring enclosing magnetic flux

Before presenting the results for uniform dephasing in the open ordered ring threaded by a magnetic flux ϕ , we first try to address the issue of why we require an extension of Büttiker's single probe model, apart from the construction of a more realistic microscopic model. In this section we work out all the results for a symmetric open ordered ring, i.e. the number of sites in the two arms of the ring between two contacts at 1 and M , are equal, or $N_1 = N_2$. All the results remain unchanged for the asymmetric case from the physics point of view. Also we keep ideal leads at 1 and M , i.e. $\gamma'_1 = \gamma'_M = \gamma$. We take a single Büttiker voltage probe and insert it in two positions of the open ring, once in the bulk of the arms between the two contacts and then at the boundary of the arms. Next the chemical potential of this voltage probe is determined from the self-consistent condition of zero average electron current from this probe to the ring. From equation (2.6) we set $j_{r-l} = 0$, where l is the position of the Büttiker probe. Then the equation is solved numerically for the chemical potential of the self-consistent reservoir with local density of states and total Green's function as given in the appendix. Finally we calculate the conductance $G(\phi)$ between two contacts at 1 and M from the same equation (2.6) for j_{r-l} but with $l = 1$ or M . In figure 2 we plot $G(\phi)$ with changing magnetic flux for two different positions of the Büttiker probe in the bulk or boundary of the arms of the open ring. In both cases coupling γ' of the probe

with the ring is the same. Though conductance profiles for the two above stated cases are not much different qualitatively, a single probe still dephases almost twice as much when in the boundary as in the bulk. So there is distinct non-universality in the results from the context of the quantity of dephasing with a single Büttiker probe depending on its position in the ring.

Now we work out the extended Büttiker model with all the sites between contacts 1 and M being coupled to side reservoirs to simulate the other degrees of freedom present in a real ring. Again to obtain the chemical potentials of the side reservoirs we fix the average electron current from these reservoirs to the ring to be zero independently. So we solve the following $N - 2$ linear equations for $N - 2$ unknown chemical potentials $\{\mu_l\}$,

$$j_{r-l} = 0 \quad \text{for } l = 2, 3, \dots, M-1, M+1, \dots, N. \quad (3.1)$$

Once the chemical potential profile of the side reservoirs is found, we use equation (2.6), with $l = 1$ or M , to determine the electron current from the source to the drain. First, we carry out both the above jobs numerically. In all the numerical results presented in this paper we set the electrical charge and Planck constant \hbar as unity. In figure 3 we plot conductance $G(\phi)$ as a function of enclosed magnetic flux for different values of the coupling γ' of the side reservoirs with the ring. Here we define conductance as the total current from the source to the drain divided by the chemical potential difference between them, $\Delta\mu = \mu_R - \mu_L$. Clearly AB oscillations of conductance $G(\phi)$ decay with increasing decoherence parameter γ' , indicating dephasing. Also the introduction of uniform dephasing does not destroy Onsager's reciprocity relation, i.e. $G(\phi) = G(-\phi)$. Using the similarity between different terms of the full Green's function and $G_{lm}^+(\omega)|_\phi = G_{ml}^+(\omega)|_{-\phi}$, we can verify that under flux reversal the solutions of equations (3.1) transform as

$$\mu_l(\phi) = \mu_1 + \mu_M - \mu_{l'}(-\phi) \quad \text{for } 1 < l < M, \quad (3.2)$$

$$\mu_l(\phi) = \mu_1 + \mu_M - \mu_{N+l'}(-\phi) \quad \text{for } M < l < N, \quad (3.3)$$

where $l' = M + 1 - l$. With these transformations and the above mentioned Green's function properties, we see that the total current, i.e. conductance, remains invariant under $\phi \rightarrow -\phi$.

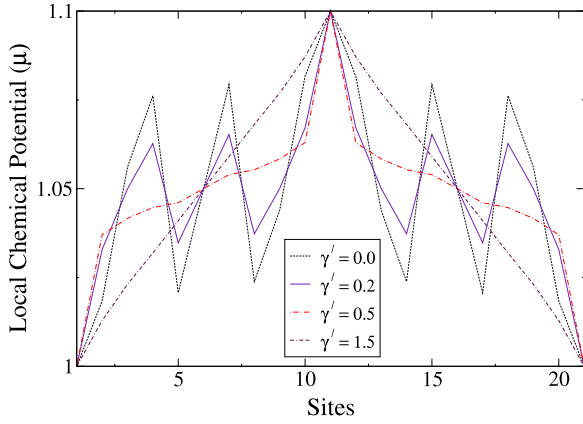


Figure 4. Plot of the local chemical potential profiles of the ring sites for different values of the decoherence parameter γ' with ϕ tending to zero, $N = 20$ and site 21 \equiv site 1.

As discussed earlier in the introduction, one elegant outcome of this extension is that we can now evaluate local chemical potential profiles of the ring's sites with changing magnetic flux by tuning γ' tending to zero. This shows analogies to a four-probe measurement of a voltage drop in a nanoscale system [15]. First, in figure 4, we give solutions of the chemical potentials from equations (3.1) with magnetic flux (ϕ) tending to zero. It shows large oscillations in the local chemical potential profile for small γ' that become flatter and flatter with increasing γ' . Finally the profile becomes completely linear for large γ' , signalling Ohmic incoherent transport of electrons in this regime, which has been discussed in great detail in our earlier work [20]. The oscillations in the local chemical potential profile for tiny decoherence can be argued to be due to the periodic geometry of the ring. A electron wave incident from the right lead gives two contributions to the current of the middle voltage probe measuring local chemical potential. First there is direct transmission into the probe, and secondly a portion of the carriers which are transmitted past the left lead by travelling through the other arm of the ring enter the voltage probe. It is the superposition of these two interfering electron waves which determines transmission in the voltage probe. Following Büttiker [21, 22] we call this *phase-sensitive* voltage measurement. For slightly larger dephasing, the flat behaviour of the chemical potential profile in the bulk of the arms and jumps at the contacts is a signature of an intermediate regime between ballistic and Ohmic transport. This pattern was quite nicely explained using a simple persistent random walk model in our previous paper [20]. In figure 5 local chemical potential profiles of the ring with changing magnetic flux ϕ are given for the completely coherent case ($\gamma' = 0$). For ϕ equal to an integer multiple of ϕ_0 , the chemical potential profiles are the same. Again for ϕ an integer multiple of $\phi_0/2$, the chemical potential profiles are similar. In both cases, the profiles are symmetric (mirror) about the contacts for the symmetric ring.

Now we derive an analytical expression for the *phase-sensitive* local chemical potential profile [23] of the ring sites with changing magnetic flux as in figure 5. We couple a single Büttiker probe invasively (though the final result is insensitive

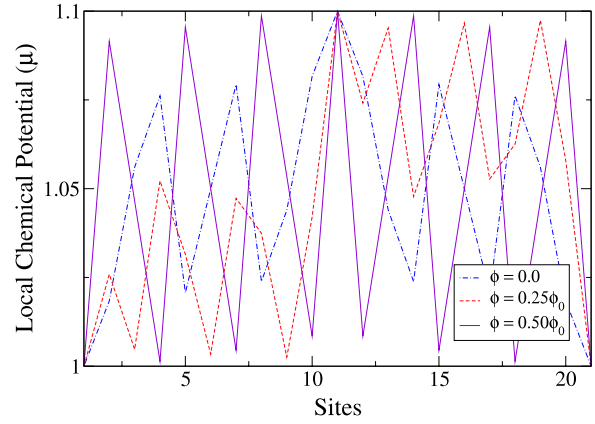


Figure 5. Plot of the local chemical potential profiles of the ring sites for different magnetic flux with γ' tending to zero, $N = 20$ and site 21 \equiv site 1.

to the coupling strength γ') with a middle site of the open ring. We then determine the chemical potential (μ_l) of the probe, i.e. the corresponding site, from the self-consistent equation (2.6). Moving the probe over all middle sites of the ring we can evaluate the full $\{\mu_l\}$ profile in a compact form:

$$\mu_l = \frac{|G_{l1}^+|^2 \mu_L + |G_{lM}^+|^2 \mu_R}{|G_{l1}^+|^2 + |G_{lM}^+|^2} \quad \text{for } l = 2, 3, \dots, M-1, M+1, \dots, N, \quad (3.4)$$

where G_{l1}^+ and G_{lM}^+ are given in the appendix. This derivation will not work for a $\{\mu_l\}$ profile with uniform finite decoherence. The oscillations in the $\{\mu_l\}$ profile depend on the Fermi energy and the applied magnetic flux through the dispersion relation.

4. Persistent current in an open asymmetric ring: the role of the eigenstates of the closed ring

In this section we investigate currents in a normal metallic ring connected with source and drain asymmetrically, i.e. $N_1 \neq N_2$. Asymmetry is very much required to achieve persistent current or a current magnification effect in the open ring in the absence of a magnetic flux. We find an analytical expression for conductance $G(\phi)$ between two contacts of the ring from equation (2.6) by evaluating the Green's function as given in the appendix. To find persistent current or circulating current in the ring, we have to know the currents in both arms of the open ring separately. First we modify equation (2.5) to get current expressions j_u and j_d in the up and down arms, respectively:

$$j_u = \frac{ie\gamma\gamma'^2}{\hbar^4} (e^{i\theta} G_{NM}^+ G_{M1}^- - e^{-i\theta} G_{1M}^+ G_{MN}^-) \times \rho(\mu_M - \mu_1) \quad (4.1)$$

$$j_d = \frac{-ie\gamma\gamma'^2}{\hbar^4} (e^{i\theta} G_{1M}^+ G_{M2}^- - e^{-i\theta} G_{2M}^+ G_{M1}^-) \times \rho(\mu_M - \mu_1) \quad (4.2)$$

with $\gamma'_1 = \gamma'_M = \gamma''$ and $\gamma' = 0$. Whenever current in an arm of the asymmetric ring becomes larger than the total current j between the source and drain a circulating current flows in

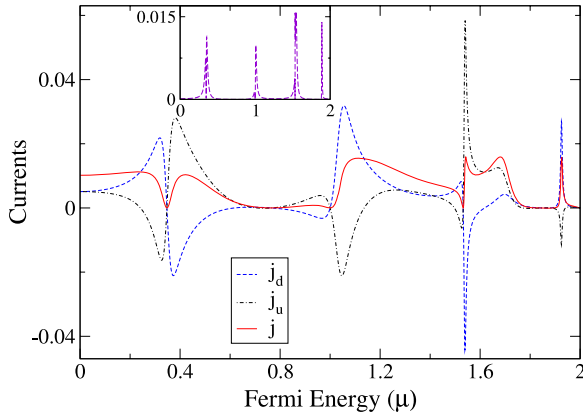


Figure 6. Plot of the currents in the two arms j_u , j_d and the total current j in the asymmetric ring with ideal leads $\gamma'' = \gamma$ in the absence of a magnetic flux ϕ . The inset shows the total current j for tunnel barriers $\gamma'' = 0.3\gamma$. In both cases $N = 18$ and $N_1:N_2 = 3:1$.

the ring that is exactly equal to the current in the other arm. This can be achieved by tuning the Fermi energy of the ring. The phenomenon of getting a larger current in the arms than the transport current is known as current magnification. The conductance $G(\phi)$ of the normal metallic ring between two contacts in the presence of magnetic flux is

$$G(\phi) = \frac{2\pi e^2 \rho^2 \gamma'^4 (-1)^{N-p} |C_{1M}^0|^2}{\hbar^5 |\Delta_N^0|^2}, \quad (4.3)$$

where p , $|C_{1M}^0|^2$ and $|\Delta_N^0|$ are given in the appendix. $G(\phi)$ is defined as $j = G(\phi)\Delta\mu/e$. Also we find from the expressions in the appendix that $G(\phi) = G(-\phi)$. Similarly, exact expressions of the currents j_u and j_d in the up and down arms of the ring can be evaluated. These expressions are quite long and are not included here. In figure 6 we plot j_u , j_d and j with changing Fermi energy of an asymmetric ring with ideal leads $\gamma'' = \gamma$ at the two contacts in the absence of magnetic flux. Clearly for some values of Fermi energy the current flows through one arm of the open ring only whereas other arm is in a completely off condition. There are some special values of Fermi energy where the total transmission from source to drain goes to zero ($< 10^{-10}$). Around these Fermi energy values the current magnification phenomenon arises. The asymmetric pattern of total transmission around these antiresonances is referred to as the Fano line shape.

We now determine the energy eigenvalues of the closed ring from the tight-binding Hamiltonian. Energy eigenvalues are $E_m = -2\gamma \cos(2\pi m/N)$ with $m = 1, 2, \dots, N$. For $N = 18$ and $\gamma = 1$, there are eight doubly degenerate eigenvalues, ± 1.87939 , ± 1.53209 , ± 1 , ± 0.347296 and two limiting values ± 2 . We find the points of zero transmission are exactly at these doubly degenerate eigenenergies. We suspect that bound states of the total Hamiltonian exist as the transmission goes to zero in the absence of an extended state from source to drain. We will show below using an effective Hamiltonian approach that indeed there are bound states embedded in the continuum of scattering states (BIC). Also, different ratios between the arm lengths, $N_1:N_2$, do change the transmission

line shape pattern but not the antiresonance positions in the energy spectrum. In the inset of figure 6 we plot total current j as a function of Fermi energy in the weak coupling limit $\gamma'' < \gamma$. The transmission zeros at the doubly degenerate eigenvalues still survive but the two neighbouring resonances around it almost merge together and their widths get reduced though the heights remain the same. Also, radiation shifts of the positions of the resonant peaks relative to the energy eigenvalues of the closed ring are observed in this regime. In the strong coupling limit $\gamma'' > \gamma$ the antiresonance points remain fixed but the resonance peaks expand.

Effective Hamiltonian approach: Following [19] the bound states are obtained as real solutions of the equation

$$[H_r + \hbar \Sigma_L^+(\omega) + \hbar \Sigma_R^+(\omega)]|\psi\rangle = \lambda|\psi\rangle, \quad (4.4)$$

where $\Sigma_L^+(\omega)$ and $\Sigma_R^+(\omega)$ are self-energy corrections arising from the interaction of the ring with the left and right reservoirs, respectively. Eigenstates of the tight-binding closed ring are given as

$$|\varphi_m\rangle = \sqrt{\frac{2}{N}} \sum_{j=1}^N \cos\left(\frac{2\pi mj}{N}\right)|j\rangle. \quad (4.5)$$

Let us multiply $\langle\varphi_m|$ from the left of both sides of equation (4.4) and then introduce the closure relation $\sum_{n=1}^N |\varphi_n\rangle\langle\varphi_n| = 1$ for the isolated ring:

$$\begin{aligned} &\langle\varphi_m|[H_r + \hbar \Sigma_L^+(\omega) + \hbar \Sigma_R^+(\omega)] \\ &\times \sum_{n=1}^N |\varphi_n\rangle\langle\varphi_n|\psi\rangle = \lambda\langle\varphi_m|\psi\rangle. \end{aligned} \quad (4.6)$$

Using the definition of self-energies we get,

$$\begin{aligned} &\sum_{n=1}^N \langle\varphi_m|\mathcal{H}_{\text{eff}}|\varphi_n\rangle\langle\varphi_n|\psi\rangle = \lambda\langle\varphi_m|\psi\rangle \\ &\text{for } m = 1, 2, \dots, N, \end{aligned} \quad (4.7)$$

with

$$\begin{aligned} &\langle\varphi_m|\mathcal{H}_{\text{eff}}|\varphi_n\rangle = E_m \delta_{mn} + g^+(\omega) \\ &\times \left[\frac{\gamma_1^2}{\hbar} \varphi_m^*(1)\varphi_n(1) + \frac{\gamma_M^2}{\hbar} \varphi_m^*(M)\varphi_n(M) \right], \end{aligned}$$

where $\varphi_n(j) = \langle j|\varphi_n\rangle$. Equation (4.7) is a matrix eigenvalue equation with \mathcal{H}_{eff} referred to as the non-Hermitian effective Hamiltonian in S -matrix theory for transmission [24, 25]. Restricting the energy of the reservoirs' electron in the conduction band, i.e. $|\hbar\omega| < 2\gamma$, we evaluate the eigenvalues of equation (4.7) numerically. The real values of λ are precisely the doubly degenerate eigenvalues of the closed ring. So there exist bound states at the energy values of Fano antiresonances. Also a change in the strength or position of the reservoir–ring couplings does not change the real eigenvalues of \mathcal{H}_{eff} . Only the complex eigenvalues get changed. This explains why the positions of the antiresonances remain same for the different arm length ratios, or with weak or strong coupling.

5. Discussion

In the present work we have removed the sensitivity of dephasing by the external probe to its position in the bulk and the boundary of the ring's arm in the Büttiker single probe model by coupling every site of the open ring with self-consistent reservoirs. Of late the mesoscopic AB oscillations have served as a measuring device for different mechanisms of electron decoherence such as electron–electron scattering and scattering off magnetic impurities [26–28]. Our extended model will be useful for understanding those experiments where the decoherence in the ring occurs uniformly because of the interactions of conducting electrons with the other degrees of freedom present in the system. There are other perfectly valid models for uniform dephasing [14, 29]. However, our extension is closer to experiments, as here the coupling between the ring and the environment is direct and easily tunable. Recently the resistance of single walled carbon nanotubes has been studied [30] in a four-probe configuration with non-invasive voltage electrodes. It was found that the four-probe resistance fluctuates and can even become negative at cryogenic temperatures due to quantum-interference effects generated by elastic scatterers [22] in the nanotube. With recent progress in experiments with quantum rings [31] we believe that it is possible to detect the local chemical potential oscillations in the open ring as predicted in the present paper. Here we should mention that differences between *phase-sensitive* and *phase-insensitive* measurements are drastic for an effectively single channel transmission problem compared to a multichannel conductor, where it depends on the particular arrangement of probe coupling [22]. Further work is also required to investigate effects of static disorder (elastic scatterer) and electron–electron interaction on the local chemical potential oscillations. There is good scope for studying the mutual effects of disorder and dissipation in dissipative open quantum systems by introducing disorder in the ring Hamiltonian through our extended model in the quantum Langevin equation approach.

The Fano antiresonance occurs because of the interference of a discrete autoionized state with a continuum. Here we have shown for single channel transport in an asymmetric open ring that the antiresonances occur exactly at the doubly degenerate energy eigenstates of the closed ring in the absence of evanescent modes. Also by finding the real eigenvalues of the non-Hermitian effective Hamiltonian we predict the existence of a BIC at these Fano antiresonance energy values. Recently some more studies have found a BIC in an AB ring and in a double cavity electron waveguide [32, 33]. Here we emphasize that in case of single channel transport the total transmission of an open symmetric ring never goes to zero in the absence of magnetic flux. Finally bound states do not contribute directly to the transport for non-interacting systems. However, as suggested by a mean field calculation in [19], they may affect the current by affecting the local density in the presence of electron–electron interactions. It will be interesting to see how the interactions between the electrons affect the transmission zeros in an open asymmetric ring.

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Appendix. Evaluation of Green's function

The full Green's function is given as $G_{lm}^+ = (\hbar/\gamma)Z_{lm}^{-1}$ where Z is a near circulant matrix with off-diagonal terms $Z_{N1} = Z_{l\ l+1} = e^{-i\theta}$ for $l = 1, 2 \dots N-1$ and $Z_{1N} = Z_{l-1\ l} = e^{i\theta}$ for $l = 2, 3 \dots N$. The diagonal terms are given by:

$$Z_{11} = Z_{MM}^+ = A(\omega) = \frac{\hbar}{\gamma} \left[\omega - \frac{\gamma'^2}{\hbar^2} g^+(\omega) \right]$$

with $\gamma'_1 = \gamma'_M = \gamma''$,

$$Z_{ll} = B(\omega) = \frac{\hbar}{\gamma} \left[\omega - \frac{\gamma'^2}{\hbar^2} g^+(\omega) \right]$$

for $l = 2, 3 \dots M-1, M+1 \dots N$.

Now using the method of [20] to determine the inverse and determinant of the tri-diagonal matrix, we can find the required inverse and determinant of the near circulant matrix Z through simple but tedious algebra:

$$\Delta_N = ((A - 2 \cosh \alpha)^2 (\cosh[N\alpha] - \cosh[p\alpha]) - 4 \sinh^2 \alpha ((-1)^N \cos[N\theta] - \cosh[N\alpha]) + 4 \sinh \alpha \sinh[N\alpha] (A - 2 \cosh \alpha)) / (2 \sinh^2 \alpha)$$

with $e^{\pm\alpha} = \frac{B}{2} \pm \left(\frac{B^2}{4} - 1 \right)^{1/2}$

with $p = N_2 - N_1$. Similarly, the co-factor can be evaluated following the above trick. Here we first find C_{1M} and calculate $|C_{1M}|^2$ which is relevant to determine the conductance $G(\phi)$ of the asymmetric ring between the drain and source contacts:

$$|C_{1M}|^2 = 2[\cosh N\alpha_R \cosh p\alpha_R - \cos N\alpha_I \cos p\alpha_I + (-1)^N \{\cos N\theta (\cos p\alpha_I \cosh N\alpha_R - \cos N\alpha_I \cosh p\alpha_R) + \sin N\theta (\sin p\alpha_I \sinh N\alpha_R - \sin N\alpha_I \sinh p\alpha_R)\}] / (\cosh 2\alpha_R - \cos 2\alpha_I),$$

where α_R and α_I are, respectively, real and imaginary parts of α . For $\gamma' = 0$, the real part of α vanishes and the coefficient of $\sin N\theta$ in $|C_{1M}|^2$ also disappears. We denote, $|C_{1M}|^2_{\gamma'=0}$ by $|C_{1M}^0|^2$ and $|\Delta_N|_{\gamma'=0}$ by $|\Delta_N^0|$.

Finally we evaluate the Green's function of equation (3.4), where a single Büttiker probe is coupled to a middle site (l) of the open ring. Here again $Z_{11} = Z_{MM}^+ = A(\omega)$, but all other diagonal terms are $\hbar\omega/\gamma$ except $Z_{ll} = B(\omega)$. The off-diagonal terms remain the same as before. Following the above method we calculate the Green's function ($l < M$)

$$G_{ll}^+ = \frac{(-1)^{l+1} \hbar}{2\gamma \Delta'_N \sinh^2 \alpha'} [e^{i(l-1)\theta} \{B(\cosh[(N-l+1)\alpha'] - \cosh[(r+1)\alpha']) - 2 \cosh[(N-l)\alpha'] + \cosh[r\alpha']\}]$$

$$\begin{aligned}
& + \cosh[(r+2)\alpha'] + (-1)^N e^{i(l-N-1)\theta} (\cosh[l\alpha']) \\
& - \cosh[(l-2)\alpha']) \\
& \text{with } e^{\pm\alpha'} = \frac{\hbar\omega}{2\gamma} \pm \left(\frac{\hbar^2\omega^2}{16\gamma^2} - 1 \right)^{1/2}, \quad (\text{A.4})
\end{aligned}$$

where $r = N - 2M + l$. In this case, we do not need to determine Δ'_N , the determinant of Z , as it gets cancelled in equation (3.4). Similarly G_{IM}^+ can be evaluated.

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